On Page 14, please replace the paragraphs beginning on Lines 17 and 27 with:

After the governing equations are formulated at step 40 a cost function is assigned at step 42. Assuming that all items in A are assigned and following standard approaches for track-to-track association, the Gaussian probability density for a given assignment set is given by:

$$P_a = \frac{e^{-\bar{x}^T R^{-1}\bar{x}/2}}{(2\pi)^{M/2}\sqrt{|R|}} \prod_i^m \frac{e^{-[A_i - B_{a(i)} - \bar{x}]^T (P_i + Q_{a(i)})^{-1}[A_i - B_{a(i)} - \bar{x}]/2}}{(2\pi)^{M/2}\sqrt{|P_i + Q_{a(i)}|}} \tag{6}$$

where $\bar{x}$ is an estimate of the relative bias . $\bar{x}_A - \bar{x}_B$ A term referred to as an assignment gate is utilized to account for unassigned elements in *a* and taking the negative logarithm of both sides and multiplying by Equation 2 yields the hypothesis score (J_a) to be maximized through choice of *a*:

$$\begin{aligned} &\delta x_i = A_i - B_{a(i)} - \bar{x} \\ &S_i = P_i + Q_{a(i)} \\ &J_a = -\bar{x}^T R^{-1}\bar{x} - \ln\left[(2\pi)^M |R|\right] - \sum_{i=1}^m \begin{Bmatrix} \delta x_i^T S_i^{-1}\delta x_i + \ln\left[|S_i|\right] & a(i) \neq 0 \\ g & a(i) = 0 \end{Bmatrix} \end{aligned} \tag{7}$$

Equation 7 assumes at least one assignment in a, and the value g, which used in deciding to accept a given assignment: g includes the missing *Mln(2n)* term and is described in greater detail below. The bias estimate, x, is chosen to maximize the assignment score, represented by J_a. The value for this is determined by taking the partial derivative of equation 7 with respect to x, setting the result to zero, and solving, resulting in:

$$\bar{x} = \left(\sum_{i=1}^{m} \begin{Bmatrix} [A_i - B_{a_i}][P_i + Q_{a_i}]^{-1} & a(i) \neq 0 \\ 0 & a(i) = 0 \end{Bmatrix} \right) \left(R^{-1} + \sum_{i=1}^{m} [P_i + Q_{a_i}]^{-1} \right)^{-1} \tag{8}$$

Again, the bias is computed based upon the assigned elements only. With no assignments made, the bias is indeterminate. Equations 7 and 8 utilize a simple bias, but could be easily reformulated with a functional bias, such as a hyperbolic displacement as a function of relative position. Other types of bias that may be accounted for include scale factor errors and rotation errors. Equations 7 and 8 together define the assignment score function that a GNP assignment algorithm must maximize. A simplification is available in the special case of homogeneous error variances (e.g., all *P1* = P -and all Qj =Q.) In this case, equation 8 becomes:

$$\bar{x}_{cv} = \left[n_a I_M + (P+Q)R^{-1} \right]^{-1} \sum_{i=1}^{m} \begin{Bmatrix} A_i - B_{a_i} & a(i) \neq 0 \\ 0 & a(i) = 0 \end{Bmatrix} \tag{9}$$

n_a : number of non-zero entries in a

It is noteworthy that the costs in equation 7 do not support generation of a cost matrix as in the classic GNN problem. Rather, the cost of any particular assignment *a(i)=j* is dependent upon the bias estimate and hence upon the entire assignment hypothesis. The entire assignment hypothesis refers to a hypothesized set of all assignments. The integer programming methods used to solve the GNN problem are based upon independent costs for each assignment and hence incapable of handling the GNP problem. This is a feature of the GNP problem that utilizes the teachings of a new invention for solution.

An "equivalent" cost function for the GNN problem is defined here. This can be a rough equivalent only, as

the bias is ignored (the assumption being that the bias is zero). The GNN formulation is in fact found by removing the bias related terms from equation 7, with result in:

$$J_{gnn} = -\sum_{i=1}^{m} \begin{cases} \left[A_i - B_{a(i)}\right]^T S_i^{-1}\left[A_i - B_{a(i)}\right] + \ln\left[\left|S_i\right|\right] & a(i) \neq 0 \\ g & a(i) = 0 \end{cases} \tag{10}$$

In general, the residual covariance Si in equation 10 would be inflated to account for residual bias errors.

After assignment of a cost function at step 42, gating is accounted for at step 44. Gating refers to accounting for non-assignment of an observation, such as observation 22 to an observation in second system 14. For any potential assignment $a(i) = j$, there are two hypotheses:

H_0*: *A_i and B_j represent independent observations and a(i) = j should be rejected in favor of a(i) = 0.

H_1*: *A_i and B_j represent the same object and a(i) = j should be accepted.

The gate value g is used in the above test. Given the interdependence of all assignments in a hypothesis, a cost cannot be uniquely defined for any particular assignment. The gating approach presented here treats the change in score when an assignment is added to a set as the cost of that assignment, and hence the value tested against g to choose H_o or H_1.

In the case where system A sends its full set of observations, the standard maximum likelihood gate used for the GNN problem is applicable but needs adjustment to allow for the extra term in equation 7 related to the

bias. Given true target density over the surveillance volume β_t, false target densities for systems A and B are β_{FTA} and β_{FTB}, and probabilities for A and B of observing a target are P_{TA} and P_{TB}, the maximum likelihood gate value is (Blackman[2], equation 9.15):

$$\begin{aligned} g &= 2\ln\left[\frac{\beta_t P_{TA} P_{TB}}{(2\pi)^{M/2} P_{NTA} P_{NTB}}\right] \\ P_{NTA} &= \beta_t P_{TB}(1-P_{TA}) + \beta_{FTB} \\ P_{NTB} &= \beta_t P_{TA}(1-P_{TB}) + \beta_{FTA} \end{aligned} \tag{11}$$

The GNP formulation is especially useful for a one-time object map handover. Typically, an object map has an a priori defined maximum number of elements, regardless of the true number of tracks present in the source. This corresponds to a lower P_{TA} and thus indicates a smaller gate value may be required than is given by equation 11.

There are m+1 terms summed to give an assignment score in equation 7 as opposed to m terms in the GNN formulation of equation 10. The extra term is due to the bias error, and as the first assignment added to a hypothesis is primarily responsible for determining the bias, the solution approach here uses $2g$ as the gating value for the first assignment. The direct impact of this decision is that entire hypotheses are less likely to be gated out given a large bias: gating out a single assignment that is marginal is acceptable, but gating out an entire hypothesis due to a large bias may not be acceptable.

After gating is accounted for at step 44, feature data may be used at step 46. The cost function given in equation 7 explicitly assumes that biases exist between the two systems in all dimensions of the common frame of reference. However, the formulation is easily expanded to include additional feature data and/or observations where the residual bias is unimportant (e.g., vehicle velocity). Assuming the feature observations are in arrays A^f and B^f with residual covariance F_{ij}, the hypothesis score becomes:

$$\delta f_i^2 = \left[A_i^f - B_{a(i)}^f\right]^T \left(F_{i,a(i)}\right)^{-1} \left[A_i^f - B_{a(i)}^f\right] + \ln\left(\left|F_{i,a(i)}\right|\right)$$

$$J_{af} = -\bar{x}^T R^{-1} \bar{x} - \ln\left[(2\pi)^M |R|\right] - \sum_{i=1}^{m} \begin{Bmatrix} \delta x_i^T S_i^{-1} \delta x_i + \ln\left[\left|S_i\right|\right] + \delta f_i^2 & a(i) \neq 0 \\ g & a(i) = 0 \end{Bmatrix} \qquad (12)$$

The score for a hypothesis at stage s, s is the number of assignment decisions made, with n_a ($n_a > 0$) assignments is:

$$J_s - \bar{x}^T R^{-1} \bar{x} - \sum_{i=1}^{s} \begin{Bmatrix} \delta x_i^T S_i^{-1} \delta x_i + \ln\left[\left|S_i\right|\right] - \ln\left(d_{\min}\right) & a(i) \neq 0 \\ \bar{g} & a(i) = 0 \end{Bmatrix} + \begin{Bmatrix} \ln\left(\left[2\pi\right]^M |R|\right) & n_a = 0 \\ 0 & n_a > 0 \end{Bmatrix} \qquad (17)$$